Proof of the equivalence of estimating the ATE under blocked assignment using inverse propensity weights and pooling the block-level estimates of the ATE, each weighted by the block’s share of the total *N*

Suppose we have *N* subjects in our subject pool. Let there be *J* blocks 1 through *J,* where *J < N*. In each block are subjects, of whom are randomly assigned to the treatment group and the remaining to the control group. Rearrange the observations such that the first observations in each block are in the treatment group and the remaining are in the control group. Summing over all blocks, and . Let the estimand be the average treatment effect .

Consider two estimators for estimating the ATE under blocked random assignment.

The first consists of the following steps: use the difference-in-means estimator of the ATE within each block and compute a weighted average of the block-level ATE estimates, with weights equal to the block’s share of the total subject pool. In other words, the weights in block *j* are and . The estimator may be written

. (1)

The second estimator weights each observation by , which for units assigned to treatment is the inverse of the probability of being assigned to treatment and for units assigned to control the inverse of the probability of being assigned to control. This probability and associated weight may vary by block. If we order the dataset so that that first units are in treatment and the remaining units are in control, this weighting scheme has the property that

. (This property only holds under block random assignment; under simple or complete random assignment, the sum of the weights will vary with the random assignment, and this equality will only be approximate.)

The second estimator may be expressed as:

. (2)

In order to see the equivalence of the two formulas, notice that for all units in the treatment group in block , the weight . Similarly, for all units in the control group in block , the weight . Substituting for , we may express equation (2) as a sum over each of the blocks:

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. (3)

which is the same as equation (1).